E1-A

SPhL Grand Finals

Sketchy Circuits (100 points)

This experimental problem was made possible with the help of SUTD.

Section A. Drawing Resistors (55 points)

Introduction

Drawing pencils work by depositing a thin layer of graphite onto paper. As graphite is electrically conductive, a pencil drawing can function as a resistive circuit. In this experiment, we will investigate the properties of such a circuit.

The resistance of a physical object is given by Pouillet's law:

$$R = \frac{\rho l}{A}$$

Where R is the resistance (in ohms, Ω), ρ is the material resistivity, l is the length of the object, and A is its cross-sectional area.

Equipment list

- **(a)** A4 printer paper sheets.
- **(b)** A4 drawing paper sheets (for use in section B).
- **(c)** 6B pencil.
- **(d)** $30\,\mathrm{cm}$ ruler.
- **(e)** Pencil sharpener.
- **(f)** Multimeter. **Read following instructions.**

Instructions

1. You may draw your resistive circuits on either the answer sheet or printer paper. If rough paper is used, indicate the problem subpart in the top right corner and submit it together with your answer sheets.
2. For all circuits, trace uniformly and steadily across the paper using the 6B pencil provided. Blow off excess graphite chips.
3. Measure the resistance by switching the multimeter dial to the resistive mode (Ω) and press one probe to each end of the resistor. There are several options; you may have to adjust the sensitivity for each reading.
4. If "OL" is shown on the multimeter (as in Figure 2 on the next page), a larger unit has to be selected. For instance, if the $20\,\Omega$ setting is selected and "OL" is shown, change the setting to $2\,\mathrm{k}\Omega$ instead.

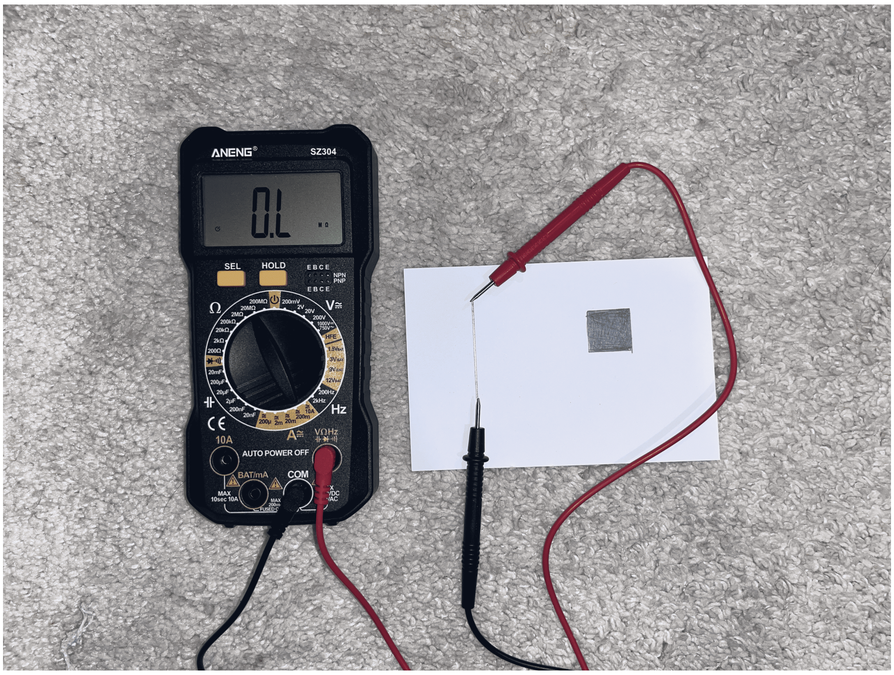
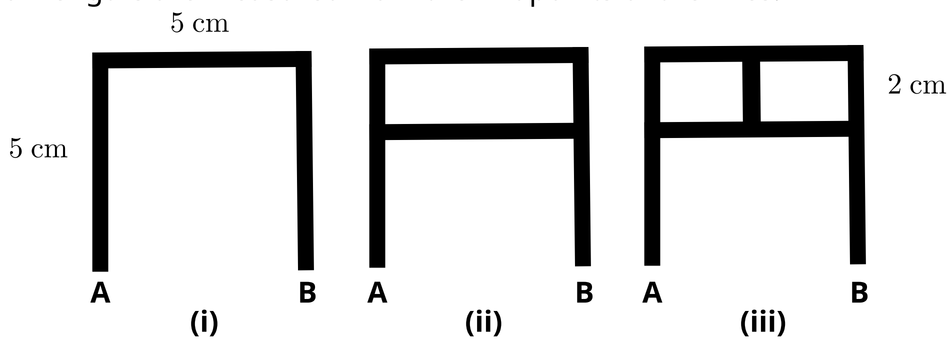


Figure 1: This picture shows the correct position of the probes when measuring resistance. Press the probes firmly to the graphite surface and adjust the precision as needed.

A.1 Draw the following three resistor networks and measure their effective resistance between points A and B. The width of each line is 0.5 cm, and all lengths are measured from the midpoints of the lines. 10 pt



Leave your answers to the appropriate number of significant figures.

A.2	Draw a single thin line with one or multiple strokes of the pencil and determine its resistance with the multimeter. By varying the length l and/or width w of the line and plotting an appropriate graph, determine the ratio ρ/t, where ρ is the resistivity of graphite and t the thickness of the graphite layer after a **single stroke** of the pencil. *Leave your final answer to the appropriate number of significant figures. Provide your tabulated data and relevant calculations. For your graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Include best-fit lines and error bars.*	20 pt

Hint: Using a single layer of graphite may produce inconsistent results. You may assume that for the first few pencil strokes, the thickness of graphite increases approximately linearly.

A.3	There is a maximum saturation thickness of graphite t_s that can be layered onto the paper via repeated tracing – after this thickness, the pencil will slide over the trace without depositing additional graphite. Determine the ratio t_s/t. *Briefly provide any procedure, argument and data collected. Leave your final answer to the appropriate number of significant figures.*	5 pt

A.4	Using any **experimental** method, determine the thickness t of the graphite layer after one stroke. Hence or otherwise, determine the resistivity ρ of graphite. *Briefly provide any procedure, argument and data collected. Leave your final answer to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.*	10 pt

A.5	Draw a resistor network between 3 points A, B and C on the answer sheet with the following properties: 1. The effective resistance between points A and B is 0.5 $\mathrm{M\Omega}$. 2. The effective resistance between points B and C is 0.7 $\mathrm{M\Omega}$. 3. The effective resistance between points A and C is 0.9 $\mathrm{M\Omega}$. *Draw your circuit on the answer sheet and label each point clearly. Points will be given for accuracy.*	10 pt

Section B. Shading Capacitors (45 points)

Introduction

A capacitor stores electrical energy using two conductive surfaces (plates) separated by a thin insulating material (dielectric). We will use pencil graphite as our capacitor plates and paper as the dielectric.

The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Where C is the capacitance (in farads, F), ε is the dielectric constant of the dielectric, ε_0 is the vacuum permittivity, A is the area of each plate, and d is the distance between the plates.

Instructions

1. For subparts B1, B2, and B4, use the drawing paper provided. Indicate the problem subpart in the top right corner and submit your circuits with the answer sheets.
2. Shade a $3\,\mathrm{cm}$ by $3\,\mathrm{cm}$ square on both sides of a sheet of drawing paper, ensuring the squares are aligned. Each square functions as a capacitor plate.
3. Measure the capacitance by turning your multimeter dial to the capacitive mode ($20\,\mathrm{nF}$ is recommended) and press one probe to each capacitor plate.
4. Ensure capacitor plates are shaded as darkly as possible to minimise resistance, and that there are no tears/cuts in the capacitor region to prevent current leakage.

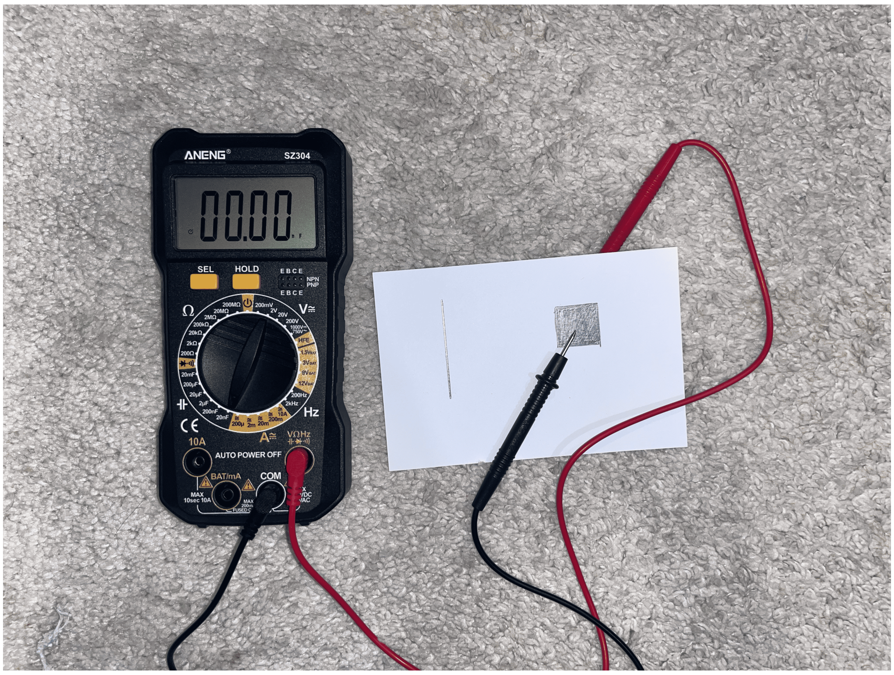


Figure 2: This diagram shows the correct position of the probes when measuring capacitance. Press the probes firmly to the graphite surface and adjust the precision as needed.

You may use $t_d = 0.25 \pm 0.02\,\mathrm{mm}$ as the thickness of a sheet of drawing paper.

B.1	Vary the shaded area and plot a graph of capacitance against plate area. *Provide your tabulated data. For your graph, draw on the graph paper provided, labelling the graph on the top right corner. Include relevant best-fit lines and error bars.*	20 pt

B.2	Hence or otherwise, estimate the dielectric constant ε_r of the paper. *Leave your answer to the appropriate number of significant figures, including propagated error. Provide all relevant calculations.*	5 pt

B.3	Shade a $3\,\mathrm{cm}$ by $3\,\mathrm{cm}$ capacitor on a sheet of *printer paper* and measure its capacitance. Explain any results qualitatively. Hence or otherwise, estimate the thickness t_p of a sheet of printer paper. *Briefly provide any procedure, argument and data collected. Leave your final answer to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.*	10 pt

B.4	Draw an RC circuit between two points A and B on a sheet of drawing paper that has a time constant of $\tau = 1.00\,\mathrm{ms}$ across AB. **Points A and B must lie on the same side of the paper.** *Provide any relevant measurements and calculations. Draw your circuit on the drawing paper provided, labelling each point clearly. The time constant will be measured and points will be given for accuracy.*	10 pt

E2-A
SPhL Grand Finals

Et Moi (100 points)

This problem was made possible with the help of SUTD.

This problem has two sections, each of which will be done separately (with different experimental equipment). For each section, you will only have its equipment for **50 minutes** — plan your time wisely!

Section A. Wheels On The Bus Go Round And Round (50 points)

Introduction

In this experiment, we will investigate the rotational inertia of a bicycle wheel. Inertia is the tendency of an object to resist any change to its state of rest or motion.

For linear motion, a net force F causes an acceleration a according to $F = ma$, where the mass, m, is a measure of the object's linear inertia.

For rotational motion, a net torque τ causes an angular acceleration α according to $\tau = I\alpha$, where the moment of inertia, I, is a measure of the object's rotational inertia.

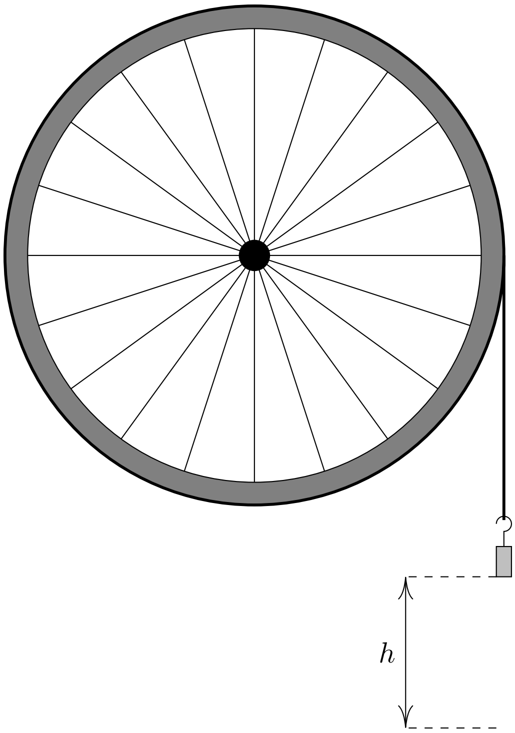
Equipment list

- **(a)** A bicycle wheel
- **(b)** 1 $50\,\mathrm{g}$ mass holder
- **(c)** 3 $50\,\mathrm{g}$ weights and 2 $10\,\mathrm{g}$ weights
- **(d)** $1.5\,\mathrm{m}$ of thick string
- **(e)** Scissors
- **(f)** Measuring tape
- **(g)** Stopwatch
- **(h)** Scotch tape

Setup Instructions

1. Secure one end of the string to the rim of the bicycle wheel by tying it to the hole located along the rim.
2. Tie the other end of the string to the mass holder.
3. Wrap the string around the rim of the bicycle wheel a sufficient number of times.
4. Load masses onto the mass holder such that it carries a total mass m.
5. Hold the bicycle wheel such that its axis is horizontal and release the mass holder from rest, such that the string is initially taut.
6. Using a stopwatch, measure the time T that the mass holder takes to fall a distance h.

Note: Be careful while handling the scissors and when dropping the weight.



Denoting the radius of the bicycle wheel as R and the gravitational acceleration as g, and assuming that the axle of the bicycle wheel is frictionless, the moment of inertia of the bicycle wheel can then be calculated according to

$$I = mR^2 \left(\frac{gT^2}{2h} - 1 \right).$$

We can obtain the 'linearised' form of such an equation, where plotting one variable (or combination of variables) against another gives the desired quantity from the gradient or intercept of the best-fit line. For example, if we know that some quantities x, y, and c are related by $c = xy$ and we wish to obtain a value of c from measuring x and y, then we can rearrange the equation to

$$y = c \left(\frac{1}{x} \right).$$

A plot of y against $1/x$ then produces a straight line, with c as its gradient. We will leave it to your team to find an appropriate linearised form of the given equation for the next part.

A.1	By varying either the total mass on the mass holder m or the height h and plotting an appropriate **linearised** graph, determine the moment of inertia of the bicycle wheel I. *Leave your final answer to the appropriate number of significant figures. Provide your tabulated data and relevant calculations. For your graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Include best-fit lines and error bars.*	20 pt

In reality, the axle of the bicycle wheel exerts a small frictional torque τ_f which counteracts its rotation. The kinetic and static frictional torques are different, which we will denote as $\tau_{f,k}$ and $\tau_{f,s}$ respectively.

A.2 Using any **experimental** method(s), measure the value of $\tau_{f,k}$ as accurately and precisely as possible. 15 pt

Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answers to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

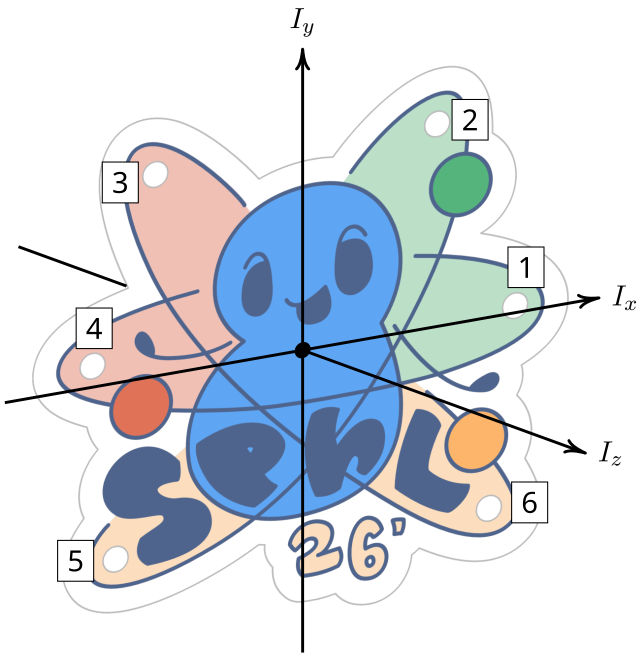
A.3 Using any **experimental** method(s), measure the values of $\tau_{f,s}$ as accurately and precisely as possible. 15 pt

Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answers to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

Section B. Bread Tastes Better Than Key (50 points)

Introduction

If you were wondering why your SPhL 2026 keychain came with several holes in it, we've decided to use it for another inertia experiment!



A bifilar pendulum is a setup where an object is suspended by two parallel strings. When the object is twisted slightly and released, it rotates back and forth. By measuring how long each oscillation takes, you can calculate the object's moment of inertia.

Equipment list

- **(a)** Retort stand with clamp
- **(b)** Sewing thread
- **(c)** Scissors
- **(d)** SPhL 2026 keychain (chain removed)
- **(e)** Measuring tape
- **(f)** Stopwatch
- **(g)** Scotch tape

B.1 Using any **experimental** method(s), determine the position of the centre of mass of the keychain. 10 pt

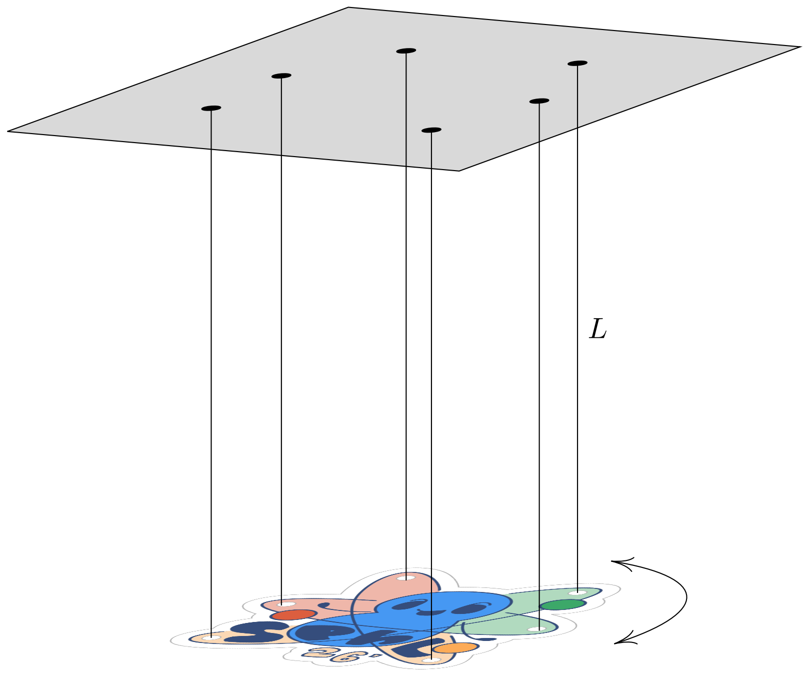
Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answer to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

E2-B

The instructions to set up a bifilar pendulum are as follows.

Setup Instructions

1. Attach one end of a string to a hole on the keychain, and attach the other end of the string to a retort stand clamp.
2. Repeat for as many strings as you wish the bifilar pendulum to have (at least 2). All strings should have the same length L.
3. Adjust the clamp heights and positions such that all attachment points lie at the same height, and all strings are perfectly vertical at rest. The horizontal distances from the attachment points to the centre of mass of the object should be as similar as possible.
4. Twist the keychain slightly and allow it to oscillate, recording the period of oscillations with a stopwatch.



We denote the mass of the hanging object as M and its moment of inertia (through a vertical axis passing through its centre of mass) as I. If the attachment points of the strings have an average horizontal distance to the centre of mass of the object a, then the period of oscillation is given by

$$T = 2\pi\sqrt{\frac{IL}{Mga^2}}.$$

B.2 Using bifilar pendula, determine the ratio of the moment of inertia to the mass of the keychain through through all axes marked on the diagram above: I_x/M, I_y/M, I_z/M, where M is the mass of the keychain. Clearly indicate which points were used to suspend the keychain (indicated in the diagram above) — the actual axes of rotation can deviate slighly from those indicated on the diagram, we will mark according to your chosen points. 30 pt

Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answer to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

Hint: You should suspend the object with more than 2 strings to measure I_z.

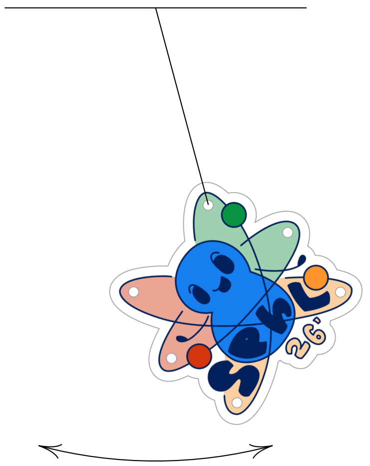
A physical pendulum is very similar to a simple pendulum, but differs in that the bob is not assumed to be a point mass, instead having a moment of inertia about its centre of mass about an axis parallel to that of the pivot I_{cm}. The mass of the bob is denoted M and the distance from the pivot to its centre of mass is denoted L. For such a pendulum, the period of small oscillations is given by

$$T = 2\pi\sqrt{\frac{I_{\text{cm}} + ML^2}{MgL}}.$$

The instructions to set up a physical pendulum are as follows.

Setup Instructions

1. Attach one end of a string to a hole on the keychain, and attach the other end of the string to a retort stand clamp.
2. Displace the keychain slightly and allow it to oscillate. Record the period of small oscillations with a stopwatch.



B.3	By using a physical pendulum, determine the ratio I_z/M of the keychain. Clearly indicate which point was used to suspend the keychain. *Leave your final answer to the appropriate number of significant figures. Provide your tabulated data and relevant calculations. For your graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Include relevant best-fit lines and error bars.*	10 pt