E1-A

SPhL Grand Finals

Stressed Out (100 points)

This experimental problem was made possible with the help of SUTD.

This problem has two sections, each of which will be done separately (with different experimental equipment). For Section A, you will only have some of its equipment for **1 hour** — plan your time wisely!

Error analysis is **not required** for this problem. Although points are awarded for the accuracy and precision of your answers, **experimental skills** carry more weightage and should be prioritised.

Section A. Bit of a Stretch (45 points)

Introduction

An elastic cord exhibits elastic behaviour by returning to its original size and shape after a load causing its deformation is removed. In this experiment, we shall investigate the elastic properties of an elastic cord.

Equipment list

- **(a)** $50\,\mathrm{cm}$ of elastic cord
- **(b)** 10 $20\,\mathrm{g}$ masses and a $50\,\mathrm{g}$ mass holder **(1 hour only)**
- **(c)** Measuring tape
- **(d)** Retort stand **(1 hour only)**
- **(e)** Scissors

Modelling

An elastic cord exerts a restoring force when stretched. Perfectly elastic objects obey Hooke's law, where the restoring force is directly proportional to its extension from rest length. However, real objects are not perfectly elastic and only obey Hooke's law for small extensions. We can account for these deviations by introducing **higher order terms**.

The restoring force generally depends on the rest length of the elastic cord, since its stiffness decreases with length. We can eliminate this dependence by using the **axial strain** to express the restoring force. The axial strain ε is defined to be the ratio between its extension ΔL and its rest length L,

$$\varepsilon = \frac{\Delta L}{L}$$

For an ideal spring, Hooke's law can be written as

$$F(\varepsilon) = \kappa\varepsilon$$

Where F is the force exerted by the spring and κ is a normalised spring constant.

For our elastic cord, to account for nonlinearity, we can model the magnitude of the restoring force F as[1]

$$F(\varepsilon) = \begin{cases} 0 & \varepsilon < 0 \\ A\varepsilon + B\varepsilon^2 & 0 \leq \varepsilon \leq \varepsilon_{\text{max}} \\ \infty & \varepsilon > \varepsilon_{\text{max}} \end{cases}$$

where A, B are constants that reflect the nature of the restoring force below its maximum length and ε_{max} is the maximum axial strain of the cord.

A.1	By making appropriate measurements, determine the maximum axial strain ε_{max} experimentally. *Briefly provide any procedure, argument and data collected. Leave your answer to the appropriate significant figures. Points will be given for the accuracy and precision of your answer.*	5 pt

Hang length L of the elastic cord on the retort stand. Ensure that the retort stand is secured firmly by its base so that it does not wobble. Then, tie the free end of the elastic cord to the mass holder and add an appropriate number of masses onto the holder to act as a load of total mass m. The extension x of the cord can then be measured.

A.2	Vary the mass of the load and/or the rest length of the cord to measure the extension. By using an appropriate linearisation, plot a linear graph to determine A and B. *Leave your final answers to the appropriate significant figures in units of* N. *Points will be given for the accuracy and precision of your answer. Provide your tabulated data and relevant calculations. For your graph, draw on the graph paper provided, labelling the graph on the top right corner. Include relevant best-fit lines.*	15 pt

The **Young's modulus** is an intrinsic property of the elastic material of the cord that describes its stiffness for small deformations (i.e. in the regime where Hooke's law applies). The Young's modulus E is defined by

$$E = \frac{\sigma}{\varepsilon}$$

where σ is the **tensile stress** of the cord, defined by

$$\sigma = \frac{F}{A_0}$$

where F is the force exerted by the material and A_0 is its initial cross-sectional area.

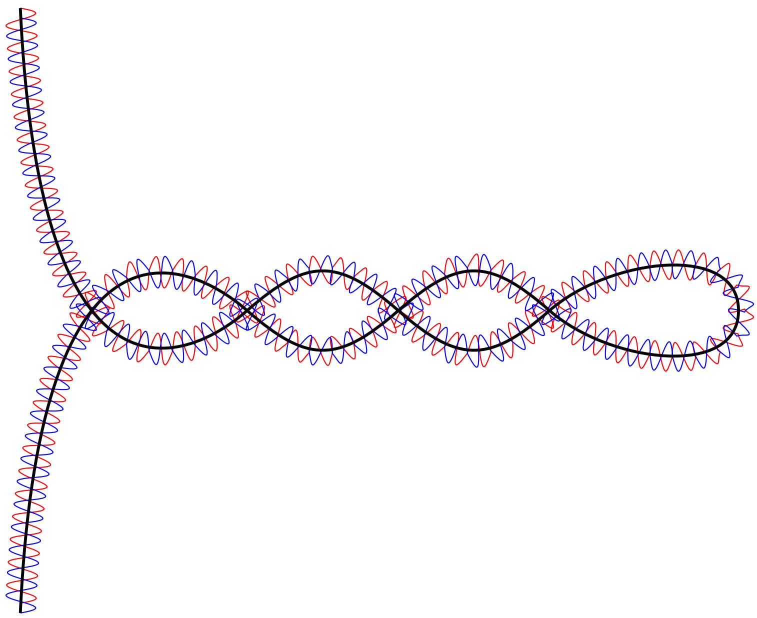
[1] In reality, the elastic cord cannot exert an infinite restoring force, but practically any load that causes its extension to go beyond the maximum length will likely snap the cord.

A.3	By making appropriate measurements, determine the Young's modulus E of the cord experimentally. *Briefly provide any procedure, argument and data collected. Leave your answer to the appropriate significant figures in units of* Pa. *Points will be given for the accuracy and precision of your answer.*	5 pt

Hint: Cutting the cord exposes the elastic material inside the cord.

The elastic cord also exhibits torsional elasticity, exerting a restoring torque with twisted. As an example, hold a section of a cord at shoulder's width and twist one end of the cord. Due to the restoring torque, you will feel an increasing resistance as the number of twists increases. However, beyond a critical number of turns, the axis of the cord will start to wind around itself in order to release some of its stored torsional energy, forming a loop called a **writhe**.

As you further increase the number of turns, the elastic cord forms additional loops out of the cord axis to release stress. This phenomenon is called **plectonemic supercoiling**. The resulting form of the elastic cord is analogous to the coiling of DNA strands.



Apply some tension T on the cord and twist the cord about one of its ends until you first observe a writhe. The number of revolutions n the cord has been twisted, which we shall call the critical twist number, can then be measured. Empirically, the critical twist number depends on the tension T in the cord as

$$n \propto T^k$$

where k is a dimensionless constant.

For the following, you may find **log-linearisation** helpful: if two quantities X, Y are related by

$$Y = X^n$$

we may take the logarithm of both sides to obtain

$$\ln Y = n \ln X$$

Then, if we plot $\ln Y$ against $\ln X$, the gradient of the graph is equal to the exponent n.

A.4 Vary the axial strain in the cord to measure the critical twist number at which plectonemic supercoiling first occurs. By using an appropriate linearisation, plot a linear graph to determine k. 15 pt

Leave your final answer to the appropriate significant figures. Points will be given for the accuracy and precision of your answer. Provide your tabulated data and relevant calculations. For your graph, draw on the graph paper provided, labelling the graph on the top right corner. Include relevant best-fit lines.

Hint: Stretching the cord with your fingers may be unreliable as your fingers will naturally compensate for the intrinsically increasing restoring force as you twist the cord. Instead, aim to use what to have to apply some known tension on the cord.

It turns out there is another, less common form of supercoiling called **toroidal supercoiling**. It also occurs when the cord is twisted, but under different conditions.

A.5 Based on your observations, draw what toroidal supercoiling looks like and state the conditions (small or large) on the length, twist number and tension of the cord that favour the formation of plectonemic and toroidal supercoiling respectively. 5 pt

Provide your diagram in the answer sheet. State clearly which phenomena correspond to which conditions under which they occur.

Section B. It's Common Ball (55 points)

Introduction

Plasticine exhibits plastic behaviour (as it name suggests) by deforming irreversibly even after a load causing deformation is removed. In this experiment, we shall investigate the plastic properties of modelling clay.

Equipment list

- **(a)** One box of modelling clay
- **(b)** Measuring tape
- **(c)** One $100\,\mathrm{g}$ mass
- **(d)** Scissors

WARNING: As the modelling clay is adhesive, please ensure that no traces of clay is left on the floor or the tables! Markers reserve the right to deduct points if great effort is required to remove any remaining traces of clay after the round.

Instructions

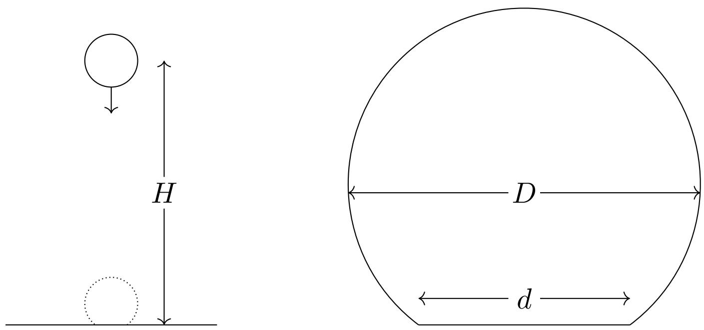
1. Use the modelling clay to create a plasticine ball of average diameter D.[2]
2. Release the ball from rest at some height H (of the centre of the ball) above the ground so that it deforms when it hits the ground.
3. Delicately pick up the deformed ball and measure the average deformation diameter d of the ball.

Modelling

An empirical model for the average deformation diameter d in terms of the average diameter of the ball D and the height H is

$$d \propto D^{\alpha} H^{\beta}$$

where α and β are dimensionless constants.



[2] The easiest way to do this is to squash the plasticine until it is roughly spherical, then roll the ball between your hands in a circular motion, slowly decreasing the force applied until the ball becomes sufficiently spherical.

B.1	By fixing the value of H and varying the value of D, and making an appropriate linearisation, plot a linear graph to determine α experimentally. You are not required to repeat measurements. *Leave your final answer to the appropriate significant figures. Points will be given for the accuracy and precision of your answer. Provide your tabulated data and relevant calculations. For your graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Include relevant best-fit lines.*	15 pt

B.2	By fixing the value of D and varying the value of H, and making an appropriate linearisation, plot a linear graph to determine α experimentally. You are not required to repeat measurements. *Leave your final answer to the appropriate significant figures. Points will be given for the accuracy and precision of your answer. Provide your tabulated data and relevant calculations. For your graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Include relevant best-fit lines.*	15 pt

Hint: You may refer to the technique used in **A.4** for these two subparts.

The deformation diameter also depends on an intrinsic property of the plasticine known as the **yield stress**. The yield stress Y is defined as

$$Y = \frac{F}{A_0}$$

where F is the minimum force required to deform the material plastically and A_0 is the area over which the force is applied.

Using the modelling clay as well as the known $100\,\mathrm{g}$ mass, we can determine the yield stress of plasticine.

B.3 Using any **experimental** method(s), determine the yield stress Y of the modelling clay. Your experiment must include at least one linear graph. You are to provide the following requirements in sufficient detail: — 25 pt

- a procedure of the steps to be taken,
- a diagram of the experimental setup with the apparatus labelled,
- a table of the data collected and the graph(s) plotted,
- an explanation for how the yield stress was determined from the data/graph(s).

*Provide your experimental write-up in the answer sheet, **labelling each requirement clearly**. Leave your final answer to the appropriate significant figures in units of Pa. Points will be given for the accuracy and precision of your method(s) and answer. Provide your tabulated data and relevant calculations in the space provided. For your graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Include relevant best-fit lines.*

Hint: Instead of measuring F directly, you can consider using the work done by F on the clay.

E2-A
SPhL Grand Finals

Laser Unfocused (100 points)

This problem was made possible with the help of SUTD.

Equipment list

- **(a)** A4 graph paper. 2 pieces have been provided, but you may approach the invigilators if you require more.
- **(b)** A4 millimeter paper. 1 piece has been provided, but you may approach the invigilators if you require more.
- **(c)** A4 protractor paper. 1 piece has been provided, but you may approach the invigilators if you require more.
- **(d)** Two lenticular sheets, with unknown radius of curvature R and line width p. One sheet is labelled "A" and the other is labelled "B". The sheets have a plastic film on their flat sides, which covers a sticky residue. We suggest that you **avoid removing this film** for the most accurate results.
- **(e)** Laser pointer of known wavelength $\lambda = 650\,\mathrm{nm}$. **Read the following instructions on how to use the laser pointer.**
- **(f)** Cardboard hair holder
- **(g)** Protractor
- **(h)** Scotch tape
- **(i)** Measuring tape

In addition, you should have the following two items with your team:

- **(a)** $30\,\mathrm{cm}$ ruler
- **(b)** Scissors

If your team does not have these items, inform the nearest invigilator immediately.

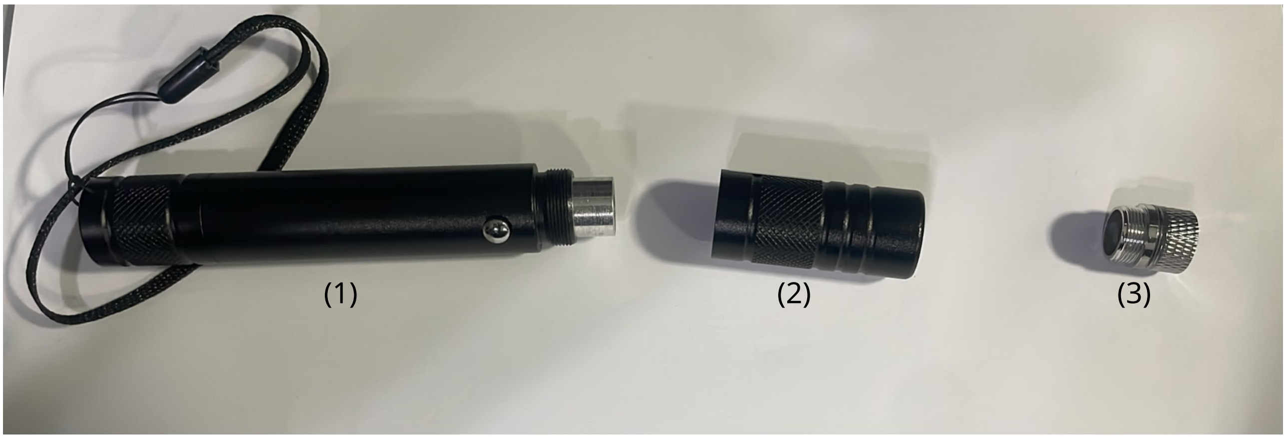
WARNING: Direct exposure to the laser beam is hazardous to the eyes. DO NOT POINT THE LASER BEAM AT YOURSELF OR OTHER PEOPLE! If you do so, you will be disqualified. Instead, please point the laser pointer *downwards* to the floor when in use. Turn off the laser when not in use as the battery is limited.

Instructions

The laser pointer consists of three separate parts that can be unscrewed as shown in the diagram below. The parts are labelled as follows:

1. (1) The collimated beam of light is emitted from this section.
2. (2) A holder for the star cap (3).
3. (3) A star cap containing the diffraction gratings that produce the pattern on the screen. This attachment can be rotated, changing the pattern on the screen.

For this experiment, you only need the star cap for A.1 through A.4. From A.5 onwards, remove the star cap.

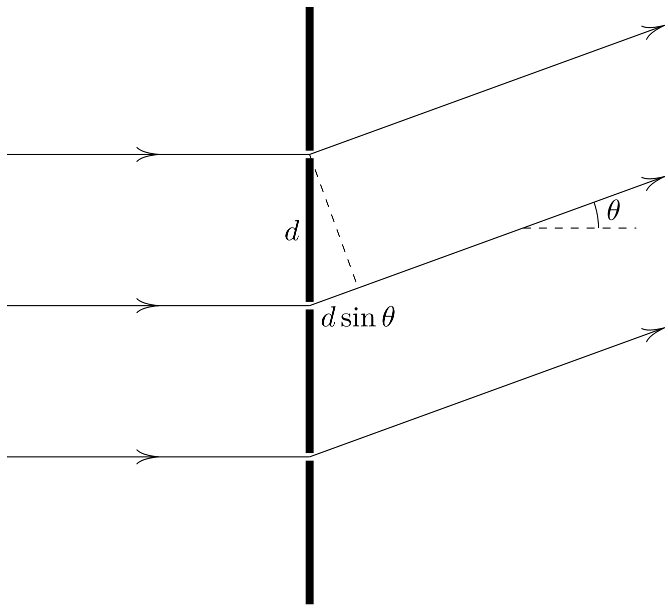


Section A. Seeing Stars (65 points)

Theoretical Background

Diffraction gratings work by exploiting the wave nature of light — the laser emits a plane wave with wavelength $\lambda = 650\ \mathrm{nm}$, which then passes through each individual slit in the diffraction grating. According to the Huygens-Fresnel principle, each slit of the diffraction grating then emits a wave spherically outwards, which can then interfere with the spherical waves from adjacent slits to form a diffraction pattern.

Since the screen on which the diffraction pattern is projected is usually much further from the grating than the distance between adjacent slits, interference between adjacent slits occurs between light exiting the slits at the same angle θ to the normal, as shown in the following diagram.



The difference in distance travelled by the 2 beams can be found through trigonometry to be $d\sin\theta$.

For constructive interference to occur, this difference in distance must correspond to an integer number of wavelengths, since this causes the peaks and troughs of adjacent beams to line up with each other, thus reinforcing the beam at that angle. We hence arrive at the following famous condition for the angles of the brightness maxima from the diffraction grating:

$$d\sin\theta = m\lambda,$$

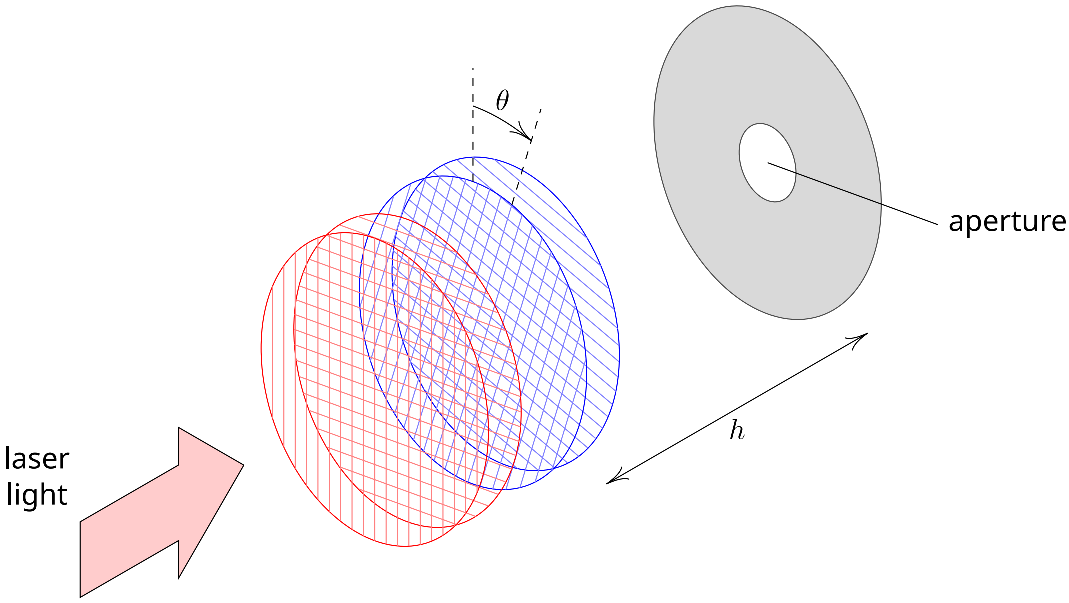
where m is an integer.

Details of the Laser Pointer

The laser pointer you are given comes with an attachment already screwed on. This attachment contains 4 identical diffraction gratings, and can be turned by twisting the star cap. The diffraction gratings come in 2 pairs, each of which consists of 2 diffraction gratings with their slits oriented at $90°$ to each other.

If two diffraction gratings are placed on top of each other such that their slits are oriented at 90° to each other, then each of the beams from the first grating is split into another series of beams by the second grating, in a direction perpendicular to that from the first grating. This causes the laser beam to be split into a 2D square grid of beams.

When 2 of these pairs are placed on top of each other, and one of the pairs is rotated by an angle θ relative to the other, each of the beams from the first pair is split into its own square grid of beams, thus forming the complex patterns that can be observed from the laser pointer.



A.1 If you look at the star cap, you will notice that the diffraction gratings are located slightly behind the aperture. Determine the distance between the diffraction gratings and the aperture h. — 10 pt

Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answers to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

A.2 Determine the distance d between adjacent slits of each diffraction grating. Show your method of measurement and working. — 10 pt

Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answers to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

Hint: When are the two gratings aligned with each other?

We can find the positions of the maxima by considering the effect of each diffraction grating separately. The outgoing angles of the beams from one grating satisfy the equation $d\theta \approx m\lambda$ (after applying the small angle approximation $\theta \ll 1$). When these beams are projected on a screen a distance L from the gratings, their distance from the central point is given by

$$a = L\tan\theta \approx m\lambda L/d.$$

A single diffraction grating produces a line of points on the screen; their positions can be written as

$$\boldsymbol{r} = i\begin{pmatrix} a \\ 0 \end{pmatrix},$$

where we use i instead of m to number the points. Following the same procedure for each of the remaining 3 diffraction gratings, we find that

$$\boldsymbol{r} = i\begin{pmatrix} a \\ 0 \end{pmatrix} + j\begin{pmatrix} 0 \\ a \end{pmatrix} + k\begin{pmatrix} a\cos\theta \\ a\sin\theta \end{pmatrix} + l\begin{pmatrix} -a\sin\theta \\ a\cos\theta \end{pmatrix}$$

When the gratings are aligned ($\theta = 0$), the maxima form a square grid with spacing D. However, at certain special angles of θ, grids with smaller spacing between maxima can be observed. The spacing between these grids D' is a fraction of D.

A.3 Find as many values of $\frac{D'}{D}$ as you can. For each value found, provide $\frac{D'}{D}$ and the rotation angle(s) θ as precisely as possible. — 15 pt

Briefly provide any procedure, argument and data collected (if any). Leave your final answers to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

A.4 Hence or otherwise, determine an **exact trigonometric form** of the rotation angle(s) θ for each measured value of $\frac{D'}{D}$ in **A.3**. For example, if you obtain an angle of $48.9°$, the exact form could be $90° - \arcsin\frac{\sqrt{45}}{10}$. — 15 pt

Briefly provide any procedure, argument and data collected (if any). Leave your answers as exact trigonometric expressions.

Finally, we will use diffraction to find the width of a human hair! For this experiment, you will require one strand of hair from a team member — we suggest using the pair of scissors which you should have brought to remove it from their scalp. You may also use the cardboard holder to secure the hair in place.

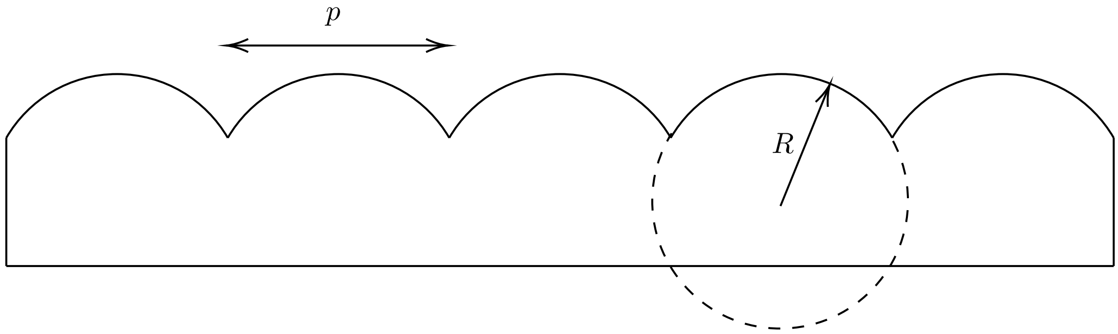
For this subpart, remove the star cap.

A.5 Determine the thickness of the strand of hair, t. 15 pt

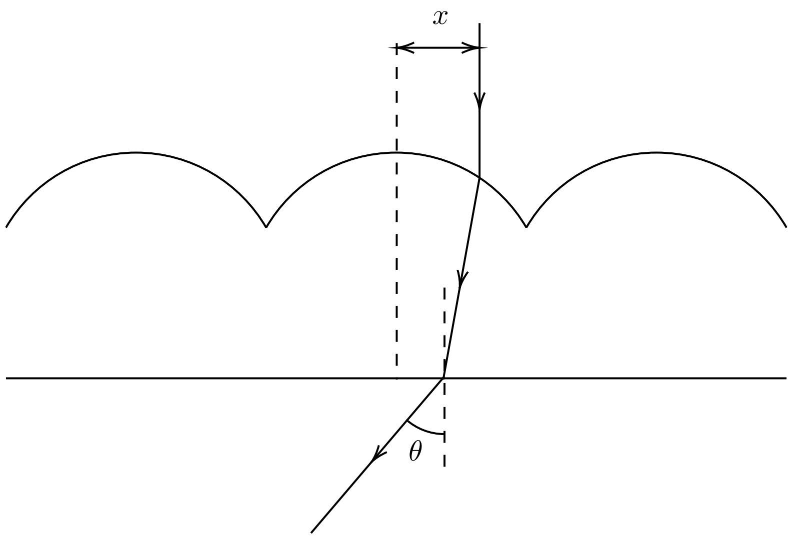
Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answers to the appropriate number of significant figures. Points will be given for the accuracy and precision of your method(s) and answers.

Section B. Seeing Double (35 points)

Your task in this section is to determine the properties of **lenticular sheets**, which consist of an array of cylindrical lenses arranged in a line. The lenses each have a radius of curvature R and the width of each lens is p. Both sheets have a refractive index $n = 1.55$. For this section, please remove the star cap from the laser.



When a light ray enters the lenticular sheet from above, it is refracted:



If the incoming light is perpendicular to the plane of the lenticular sheet and enters at a distance x from the centre of a cylindrical lens, the final angle θ is given by

$$\sin\theta = \frac{x}{R^2}\left(\sqrt{n^2R^2 - x^2} - \sqrt{R^2 - x^2}\right).$$

B.1	Determine p for sheets A and B. Label clearly on your answer sheet which values of p correspond to sheets A and B. **Error analysis is required for this part.** *Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answer to the appropriate number of significant figures, including propagated error. Points will be given for the accuracy and precision of your method(s) and answers.*	15 pt

Hint: If you shine the laser directly through the lenticular sheet and project the resultant image on a screen, a diffraction pattern can be seen.

B.2	Determine R for sheets A and B. Label clearly on your answer sheet which values of R correspond to sheets A and B. **Error analysis is required for this part.** *Briefly provide any procedure, argument and data collected (if any). For any graph(s), draw on the graph paper provided, labelling the graph on the top right corner. Leave your final answer to the appropriate number of significant figures, with propagated error. Points will be given for the accuracy and precision of your method(s) and answers.*	20 pt

Hint: You should solve this subpart by considering geometric optics, rather than diffraction.